Informational Paradox of external financing

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In this paper I develop a model for small and medium enterprises’ external financing. I concentrate on a situation where a firm offering trade credit to his customer applies for a credit at a bank. The effect of a defaulting customer on the borrowing capacity of the supplier has not yet been covered by the literature. In a contract theoretical more generally, in a game theoretical framework, there is an informational asymmetry between the lender and the borrower on the credit worthyness of the entrepreneur. In this model, the informational asymmetry leads to moral hazard which results in credit rationing. An optimal contract considering also a possibly defaulting customer generates additional credit rationing. The paper develops two subcases: in the first one the borrower has informational advantage on his customer compared to the creditor, in the second there is not any informational advantage related to the customer. The results show an informational paradox, the informational advantage of the borrower reduces the volume of credit he can receive, while the informational symmetry leads to a higher borrowing capacity. The model describes a typical situation on the Hungarian market. Banks provide a low volume of credit to SME because the firm’s financial statements and their relationship with their customers is not open. So the informational disadvantage induces the bank to offer a low level of credit even if the net present value of the financed projects is highly positive.

In this paper I examine the external financing of firms who provided trade credit to their customers. Trade credit is a widespread phenomenon. All time when a customer pays only after the delivery of the product, the supplier offered trade credit to him.

Disregarded from trade credit, external financing of firms takes place in an asymmetric informational situation where informational asymmetry applies to the firm’s the future ability and willingness to pay. Also moral hazard is considered by the financer because the creditor can not be certain of the firm’s commitment to a successful project. As a consequence, even projects with positive net present value (NPV) receive less external financing than the optimal level, if they receive funding at all. The phenomenon is called credit rationing. (Tirole, 2005)

In this paper I will demonstrate that taking trade credit into consideration, credit rationing increases. In a contract theoretical framework, based on the external financing model of Tirole (2005), I develop two alternative models. In the first version, the entrepreneur is informed about his customer’s default before the bank, while in the second version, the entrepreneur and the financer learns about delayed payments at the same time. The results show a paradox situation. In the first case the credit rationing is higher than in the
second one. So the informational advantage of the borrower on his own customer cannot be converted to higher borrowing capacity. To increase the level of external financing, the borrower should share all the information he has on his customers with the bank.

The paper organizes as follows. After reviewing the literature in Section 1, I develop the two versions of the model in Section 2. Section 3 discusses the results while Section 4 concludes.

Theoretical background


In the U.S, 16% of SME external financing is coming from the suppliers in the form of trade credit. The maturity of trade credit differs in a wide range, but the ex post collection period of these receivables became drastically longer in many European countries after the financial crises. According to Federation of Small Businesses (FSB) in the UK, receivables are collected often 60 – 125 days after the original due date. (FSB, 2009)

The results of Bartlett and Bukvic can provide an illustration for the Eastern-European region. Their survey shows that late payments are general in the Slovenian SME sector, nearly 50% of the firms mentioned overdue receivables as one of their important problems. In Hungary, according to the Institute of Economic and Enterprise Research (GVI), 36,5% of customers pays later than contracted, and 38,9% of the total sales revenue is collected after the due date. (Makó–Gyűrű–Papp [2009]).

Late payer customers often contaminate partner firms by contributing to delays on suppliers’ own payables. In Hungary, 41,9% of firms decides to delay the pay out of suppliers if their own customers do so. (Makó–Gyűrű–Papp [2009]). While an important proportion of SME are kept in queuing trade credit chains, lack of external financing is also a sever problem in the sector. This double difficulty motivated my research on borrowing capacity of firms offering trade credit to their customers.
The model of external financing in the case of defaulting customer

In this section I develop a new version of Tirole’s (2005) model for external financing. I incorporate a defaulting customer using three parameters into this game theoretical framework. The probability of collection on time is denoted by $q$, where $c$ is the proportion of the customer in the supplier’s balance sheet. The model allows a macroeconomic, systematic shock affecting both entrepreneurs, and reducing their probability of success by $\delta$, where $0 < \delta \leq 1$. The two versions of my model differ only in the following assumption. First, I assume that the maturity of trade credit is shorter than that of the bank loan, so a default of customer is already known for the supplier during the lifetime of his credit contract. The bank has to face partly the business risk of its borrower and also the related credit risk, where these two risk types lead to moral hazard. Here, an additional moral hazard appears as the borrower has informational advantage not only on his efforts to repay the loan but also on his customer’s possible default. The second version of the model defines equal maturities for the trade credit and the bank loan.

The model of external financing with relative informational advantage of the supplier

In this model there are three participants, the bank, the customer, and the supplier. The entire model is described from the point of view of the last one. The supplier has a project of seize $I \in [0, \infty)$. This variable seized initial investment yields a constant $R$ revenue on one unit investment. So in the case of success, a total revenue of $RI$ is generated, which scenario has a probability of $p$, and the revenue is zero otherwise, which event occurs at a probability of $(1-p)$. The entrepreneur’s liability is limited in the sense that he can not lose more than his initial investment.

The revenue of the project is subject to moral hazard because the supplier can decide on the level of his own efforts which also defines the probability of a successful project. The probability of success is $p_H$ if the entrepreneur „behaves”, where „behaving” means that the entrepreneur is working on the project with high efforts. But if the entrepreneur „misbehaves” or „shirks”, the probability of success is $p_L$, which is lower than $p_H$. The term „misbehaving” describes a strategy where the borrower is using the assets for his own purposes and not the generate project revenues the highest possible. In the case of misbehaving, the entrepreneur will have a private benefit of seize $B$ per unit of investment, which can be explained as the spared efforts or the gains of the private use of the firm’s assets and infrastructure.

I assume that the project has a positive NPV if the supplier behaves. As the $R$ revenue on one unit of investment is constant, it is worth to reach the highest volume possible of $I$. At the beginning of the project, the entrepreneur has only $A$ amount of cash at hand, so he has to apply for a credit of $(I-A)$. The bank requires a revenue of $R_f$ from the total revenue $RI$. The residual $R_b$ revenue is the entrepreneur’s stake in the project. The bank is facing
perfect competition, so the ex post profit of the creditor is zero. The bank is offering credit at an interest rate normalized to zero, so none of the agents of the model has a preference on the timing of revenues.

All participants are risk neutral, their decisions are made based only on the expected NPV of the different strategies. I assume that the expected NPV of the project is positive only if the borrower behaves, and negative otherwise even with the privat benefit of shirking.

In the borrower’s balance sheet, there are also receivables of seize $cI$ related to one given customer. The collection of these receivables is due before the end of the bank loan’s maturity. At a probability of $q$, the borrower collects his receivables at time, where $q$ is also known by both the supplier and by the bank ex ante. But the bank does not have any clear information on the collection ex post. If the customer defaults, the supplier’s balance sheet decreases to $(I-cI)$. I assume that even the private benefit will decrease to $BI(1-c)$ because it was originally defined to be proportional to the project seize.

A possible default of the customer has an influence also on the supplier’s probability of success. Common macroeconomic factors or industrial specialties can explain why the probability of success will be decreased by a multiplier of $0 < \delta \leq 1$. The supplier decides on his efforts after the collection of his customer.

The extensive form of the model is described in the Figure 1. The participants of the project are the bank, the supplier and the nature. In the later one I used the game theoretical solution to incorporate the realization of random variables to the model. The scenario highlighted by blue is as follows. The bank provides credit to the borrower. The customer of the borrower pays at time. The supplier decides to misbehave but even this low probability of success is enough for a successful project. So the final payout of the entire project is $IR$, that one of the borrower equals $R_b + BI$. The bank receives $(I-A)$. 

The model of external financing with symmetric information on partner risk

In the second version of the model, the borrower’s receivables are due at the end of the bank loan’s maturity which I explain by industrial differences compared to the first version. So the ex post paying ability of the supplier’s customer is not only hidden for the bank but also for the supplier. When the borrower decides on his strategy, his decision can be made only on \( q \) which is the ex ante probability of customer’s successful collection and the chosen strategy can not be modified any more. So the borrower’s and creditor’s knowledge on the borrower’s customer are symmetric.

The external form of the model is as follows in the Figure 2. The highlighted scenario describes that the bank provides credit to the supplier who decides to behave. At the due date of his receivables he collects on time the amount of \( cI \) from the customer. But even in this favorable situation the project finally fails. The project revenue and the revenue of the borrower and the creditors equals zero.
The optimal credit contract

In the above described contract theoretical framework, I define the optimal contract for the participants where the borrower has a stake in the project high enough to behave but also the bank can achieve his target revenue.

The optimal contract in the case of informational asymmetry on the customer

The total revenue of the project has to satisfy the bank’s and also the borrower’s expectations. As the decision about the credit contract is made, the ex post revenue is not known yet, the participants can only decide based on the expected values.

At first, let’s concentrate on the bank’s financial constraint. The creditor’s initial outlay is $(I-A)$ at the beginning of the project. So he requires a revenue which will not be lower than $(I-A)$. Remember, that the NPV of the project is positive only if the borrower behaves, and
negative otherwise. So the bank receives a repayment only in the former case. That’s the reason why the bank has to motivate the borrower to behave.

As the borrower’s strategy can vary according to his customer’s default or surviving, also the bank has to consider both subcases which lead to a doubled financial constraint. The bank is acting at a perfect competitive market, so (1a) and (1b) can be reduced to equations where (1b) will be the binding constraint.

\[ p_H (R - R_b) \geq I - A \quad (1a) \]
\[ \delta p_H [(1 - c)R - R_b ] \geq I - A \quad (1b) \]

The borrower’s incentive constraint has to assure the borrower’s high efforts. To induce him to behave, he has to keep a proportion of the total project revenue high enough to offset the BI private benefit of shirking. The contract has to incentive him also in the case of his customer’s default, so according to the paying ability of the customer, two subcases are to be defined.

\[ p_H R_b \geq p_L R_b + B \quad (2a) \]
\[ \delta p_H R_b \geq \delta p_L R_b + B (1 - c) \quad (2b) \]

Let \( \Delta p \) the difference of the two probabilities of success, more precisely \( \Delta p = p_H - p_L \). Substituting the borrower’s incentive constraint to the bank’s financial constraint, the optimal contract can be described as follows.

If \( 1 - c < \delta \) - let’s call that Case 1 – the optimal contract has to fulfill the following criterion:

\[ A \geq I \left\{ 1 - \delta p_H \left[ R(1 - c) - \frac{B}{\Delta p} \right] \right\} \quad (3a) \]

If \( 1 - c > \delta \) - which will be the Case 2 – the optimal contract has different characteristics:

\[ A \geq I \left\{ 1 - p_H (1 - c) \left[ R \delta - \frac{B}{\Delta p} \right] \right\} \quad (3b) \]

Having (4a-b) technical assumptions, I restate the (3a-b) inequations in formulae (5a-b).

\[ 1 - \delta p_H \left[ R(1 - c) - \frac{B}{\Delta p} \right] > 0 \quad (4a) \]
\[ 1 - p_H (1 - c) \left[ R \delta - \frac{B}{\Delta p} \right] > 0 \quad (4b) \]

\[ A \left\{ \frac{1}{1 - \delta p_H \left[ R(1 - c) - \frac{B}{\Delta p} \right]} \right\} \geq I \quad (5a) \]
Let's use notation $\mathcal{H}_i \geq I \ (i=1, 2)$ to simplify the formulae, so I can define the equity multipliers $k_1$ and $k_2$ for Case 1 and 2.

$$k_1 = \frac{1 - \delta p_H (1 - c) \left[ R \delta - \frac{B}{\Delta p} \right]}{1 - p_H (1 - c) \left[ R \delta - \frac{B}{\Delta p} \right]} > 1 \quad (6a)$$

$$k_2 = \frac{1 - \delta p_H (1 - c) \left[ R (1 - c) - \frac{B}{\Delta p} \right]}{1 - p_H (1 - c) \left[ R \delta - \frac{B}{\Delta p} \right]} > 1 \quad (6b)$$

Analyzing the (6a-b) equations, in a model where the borrower does not have defaulting customers, the denominator of the equity multiplier reduces to $1 - p_H (R - \frac{B}{\Delta p})$ as the parameters related to the customer take the following values: $\delta = 1$ and $c = 1$. In such a very simple case, the maximum seize of the revenue pledged to the bank without violating the borrower’s incentive constraint is $p_H (R - \frac{B}{\Delta p})$. The $p_H \frac{B}{\Delta p}$, the second part of the maximum pledgeable income is a kind of agency cost which is used by the bank to induce the borrower.

When the borrower can have defaulting customers, the equity multiplier is changed by the decreased pledgeable income. The maximum pledgeable income reduces not only because the expected revenue of the project decreases to $\delta p_H R (1 - c)$, but also because the decrease in the agency cost, $p_H \frac{B}{\Delta p} \delta$ or $p_H \frac{B}{\Delta p} (1 - c)$, is relatively lower compared to the decrease of the expected revenue.

The optimal contract in the case of informational symmetry on the customer

The optimal credit contract is derived also in this case from the financial constraint of the creditor and from the incentive constraint of the borrower. The risk neutral participants base their decisions again on the expected values of their future revenues.
The bank requires an expected revenue which is not lower than his initial outlay of $(I-A)$. The creditor has to motivate the borrower to behave, but in this case the moral hazard related to the borrower’s decision is lower than in the former version of the model. The borrower chooses his strategy at the beginning of the project when a possible future default of his customer is unknown. So there is only a moral hazard related to the level of entrepreneur’s effort in this version of the model, but there is not any moral hazard due to a defaulting customer. So the participants can define their own constraints in one single expected value without differentiating two subcases depending on the customer’s liquidity situation. The financial constraint of the bank and the incentive constraint of the borrower reduces to (7) and (8) compared to the former section.

\[
\begin{align*}
\Phi_H (R - R_b) + (1-q) p_H (R (1-c) - R_b) & \geq I - A \\
p_H R_b & \geq p_L R_b + B \left[ q + (1-q) (1-c) \right]
\end{align*}
\]

(7) \hspace{1cm} (8)

Using (7), (8) and the notation $\Delta p = p_H - p_L$, the optimal credit contract has the following characteristics:

\[
A \geq I \left\{ 1 - p_H (R[q + \delta(1-q) 1-c]) - \frac{B}{\Delta p} [q + (1-q) 1-c] \right\}
\]

(9)

Remembering that the NPV of the project is positive only if the borrower behaves, the right side of inequation (9) is composed of the I project seize and of a multiplier smaller than 1. Again, an additional assumption (10) is needed to finish the calculation:

\[
1 - p_H (R[q + \delta(1-q) 1-c]) - \frac{B}{\Delta p} [q + (1-q) 1-c] > 0
\]

(10)

The relationship between the borrower’s initial wealth and the project seize is as follows in (11a-c). In this version a single $k$ equity multiplier can be defined using (11c):

\[
\begin{align*}
A \geq I \left\{ 1 - p_H (R[q + \delta(1-q) 1-c]) - \frac{B}{\Delta p} [q + (1-q) 1-c] \right\}
\end{align*}
\]

(11a)

\[
\begin{align*}
\Phi_H (R - R_b) + (1-q) p_H (R (1-c) - R_b) & \geq I - A \\
p_H R_b & \geq p_L R_b + B \left[ q + (1-q) (1-c) \right]
\end{align*}
\]

(11b)

\[
\begin{align*}
k = \frac{1}{1 - p_H (R[q + \delta(1-q) 1-c]) - \frac{B}{\Delta p} [q + (1-q) 1-c]} > 1
\end{align*}
\]

(11c)
Here, in this version of the model the maximal pledgeable income decreases due to the lower expected project revenue and due to the relatively higher agency cost which is not affected by the macroeconomic shock $\delta$.

**Results**

In this section I analyze the borrowing capacity of entrepreneurs comparing the two versions of the model.

**The equity multiplier**

The optimal contract which the $k_1, k_2$ or the single $k$ multiplier is derived from also defines the maximum level of the entrepreneur’s leverage. As the borrower can keep the entire NPV of the project – because the bank requires a revenue of only $(I-A)$ – the entrepreneur is interested in a maximal project seize. Inequations (5) and (11) define the maximal seize of $I$ at a given level of entrepreneur’s $A$ initial wealth. So the optimal investment decision if the supplier is to invest $k_i (i=1; 2)$ or $k$ times his initial $A$ amount of assets where $k_i (i=1; 2)$ or $k > 1$. So the entrepreneur applies for a credit of $(k_i -1)A$ (where $i=1; 2$) or $(k-1)A$ to boost the project seize up to $I$.

The higher the $k_i (i=1; 2)$ or $k$ equity multiplier the higher the maximum seize of the project. Disregarded from defaulting customers, a high probability of success in the case of the borrower’s high efforts ($p_H$), a high project revenue per one unit of investment ($R$), a low level of private benefit of shirking ($B$) can increase the borrower’s borrowing capacity. Also a large difference between the probabilities of success of shirking and behaving ($\Delta p$) can contribute to an increased leverage, because the efforts of the borrower are more closely related to the success of the project than at a smaller $\Delta p$. A high $\Delta p$ means a higher probability of negative consequences in the case of shirking so the moral hazard is lower if the $\Delta p$ is high.

Considering also possible defaults of the borrower’s customers, the borrowing capacity of supplier decreases. The decrease can be measured by the difference between the equity multiplier in the case of $q=1$ – let denote this multiplier by $k^* -$ and the appropriate of $k, k_1, k_2$ in the case of $q<1$. Analysis of the formulae shows that the higher the proportion of the customer in the borrower’s balance sheet ($c$), the lower the level of accessible external financing. If also common macroeconomic shocks can occur or the financial stability of borrower is influenced by the default of his customer through the decreased probability of success ($\delta$), the borrowing capacity will be even lower.

In the first version of the model (informational asymmetry on the customer), the $q$ probability of the collection of receivables is not part of the equity multiplier. From the theoretical point of view, the assumptions of the model can explain the result. The practice can also provide an evident explanation which harmonizes with my results. A possible
customer default effects the bank’s exposure on the one hand through the financial stability of the borrower which decides whether a contagion can occur or not (δ), and on the other hand through the borrower’s exposure to his customer measured by c. An other reason why the bank is interested in these parameters is that the borrower decides on his strategy based on the values of c and δ. So the level of additional moral hazard related to the borrower’s customer is influenced only by c and δ but not by q. The estimation of q which could be forecasted based on the average collection period of receivables is often insufficient because financial statements of SME usually lack opacity not only in Hungary but also in more developed economies.

According to the findings, firms with less diversified customer portfolios, depending on a few strategic customers, and at the same time having a long collection period of receivables can achieve a lower level of external financing than similar firms with a more diversified portfolio. A low level of δ can also contribute to a decreased borrowing capacity disregarded from the level of c. A high dependency on timing of collections, a low volume of net working capital can lead to such a situation. Possible further development of the model could be to define δ as a function of c.

In the case of informational symmetry on the customer, the q probability of the customer’s punctual collection is also part of the results. In this version the severity (δ and c) of the customer’s default and also the q probability of his default can influence the equity multiplier. Compared to the former analysis, the c and δ has the same effects on the entrepreneur’s borrowing capacity and a decrease of q can also reduce the level of leverage.

The next step of my analysis is to quantify the decrease in the supplier’s borrowing capacity. As a benchmark, I will use k* the maximal borrowing capacity where sales revenue is always collected on time (q=1). In this case my results reduce to the model of external financing described by Tirole (2005) because the optimal contract is defined by (1a) and (2a):

\[
k^* = \frac{1}{1 - p_H(R - \frac{B}{\Delta p})}
\]

In Table 1, there are the equity multipliers for the case of informational asymmetry where \(k_1\) is for the case of \(\frac{1-c}{\delta} < 1\), and \(k_2\) describes the situation of \(\frac{1-c}{\delta} > 1\). The \(k\) is standing for the borrowing capacity if the entrepreneur does not have any informational advantage on his customer. All the three multipliers are defined as a function of \(k^*\), so one can prove that \(k_1, k_2\) and \(k\) are smaller than the benchmark \(k^*\) because denominators in the formulae are higher than 1.
Equity multipliers

Let’s choose the case of informational symmetry to illustrate why the borrowing capacity decreases compared to \( k^* \). The first part of the denominator, \((k^*-1)/(1-q)c\), is the expected loss of the bank due to the default of the borrower’s customer. The residual part of the denominator contains the expected decrease of the project’s revenue. First, the expected revenue per one unit of investment decreases by \((1-\delta)(1-c)\). Second, there is also a decrease in the expected seize of the project by \((1-c)\). So these double effect of defaulting customer is considered in the following part of \( k \):

\[
k = \frac{k^*}{k^*-(k^*-1)(1-q)c + k^2 p_H R(1-\delta)(1-c)} = \frac{k^*}{1+(k^*-1)c + k^2 p_H R(1-\delta)(1-c)}
\]

An interesting question after comparing the \( k_1 \), \( k_2 \) and \( k \) borrowing capacities to \( k^* \) is to compare the borrowing capacities of the symmetric and the asymmetric model. Having the assumptions of \( k^* > 1 \), \( 0 < q \leq 1 \); \( 0 < \delta \leq 1 \); \( 0 \leq c < 1 \) and using that \( k_1 \) is calculated by assuming \( 1-c < \delta \) and \( k_2 \) is assuming \( 1-c > \delta \), one can prove that \( k \) always provides a higher borrowing capacity than \( k_1 \) or \( k_2 \). While the model uses two subcases to define the constraints which the \( k_1 \) and \( k_2 \) are derived from, \( k \) is coming from a constraint defined by one single expected value. The fact, that the supplier can not modify his strategy after a possible default of his customer, results in a lower level of credit rationing than the other version of the model where the supplier - who is relatively well informed about the customer - can renew his decision about his level of efforts. The model shows an informational paradox: the lower informational advantage of the borrower results in a higher borrowing capacity. This paradox can be explained by the risks generated by the customer. A possibly defaulting customer means by definition an explicit credit risk for the borrower and an implicit one also for the bank. If the value of \( \delta \) is lower than one, a component of systematic risk occurs in the form of contagion between the two entrepreneurs. In the case of asymmetric information about the customer, an additional moral risk
is generated because he can change his strategy about the level of efforts. If the supplier knows relatively less about his customer, he can only take a smaller informational advantage, and the bank remunerates a relatively uninformed borrower by offering a higher credit.

Conclusions

In this paper I examined the external financing of a supplier whose customers can default. The contract theoretical framework comes from Tirole’s (2005) models on corporate finance. The customer is incorporated by three parameters to the model. The probability of customer’s default is (1-q), where the receivables related to this customer have a part c in the balance sheet of the supplier. A default of the customer or of both firms can be explained by a common macroeconomic shock, by \( \delta \). The \( \delta \) can also describe a coefficient of contagion between the two firms. But disregarded from its interpretation, \( \delta \) brings a systematic risk component to the model. If a possible default of a customer appears in the model of the supplier’s financing, the optimal credit contract of the supplier will change. The bank provides a lower level of credit that he would without the possible overdue receivables of his borrower.

The credit risk of the customer results in a lower project seize which decreases both the borrower’s NPV and the generated social wealth. The paper developed two versions of the model where difference of the two models was the level of informational asymmetry between the bank and the borrower. If there is an informational asymmetry not only about the efforts of the borrower but also about the customer’s credit risk, this allows the supplier to modify his strategy after the due date of his receivables. The customer’s credit risk which can also lead to a systematic risk component generates additional moral hazard of the supplier. The bank, considering all the three additional risk elements, reduces the volume of credit which would be already rationed without any partner risk related to the customer.

If the supplier has not any informational advantage on his own customer, he achieves a higher leverage than in the “well informed case”, what I describe as an informational paradox. In this version of the model, a possibly defaulting customer generates only credit risk and a contagion effect which the bank will be exposed to, but there is not any additional moral hazard which the supplier could take advantage from. That is the reason why the effect of the risky customer reduces relatively less the credit worthiness of his supplier than in the first version of the model.

The results of this paper shows that the borrowing capacity of SME could be ameliorated if the bank has sufficient information also on the customers of his borrower. On possible way of having all important information also on the customer is to provide bank services also to the customer himself. Such an acquisition of customers is quite a common strategy, several participants of the Hungarian bank market have the same practice. So the first empirical overview does not contradict to the theoretical results of my model. Although the diversification of the credit portfolio decreases if the participants of the same supply
chain are financed by the same bank, the level of external financing of the firms can be improved. So the direction of further research will be the joint financing of the supplier and his customer, this two neighbors in a supply chain.

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